

GOAL

SOLVE THE FOLLOWING EQUATION :

$$\frac{\text{natural deduction}}{\lambda\text{-calculus}} = \frac{\text{deep inference}}{\times}$$

natural deduction



λ -calculus



functional programming
LISP, Haskell, ML

deep inference



categorical combinators



'concatenative programming'
Postscript, JVM

natural deduction



λ -calculus



functional programming
LISP, Haskell, ML

deep inference



categorical Combinators

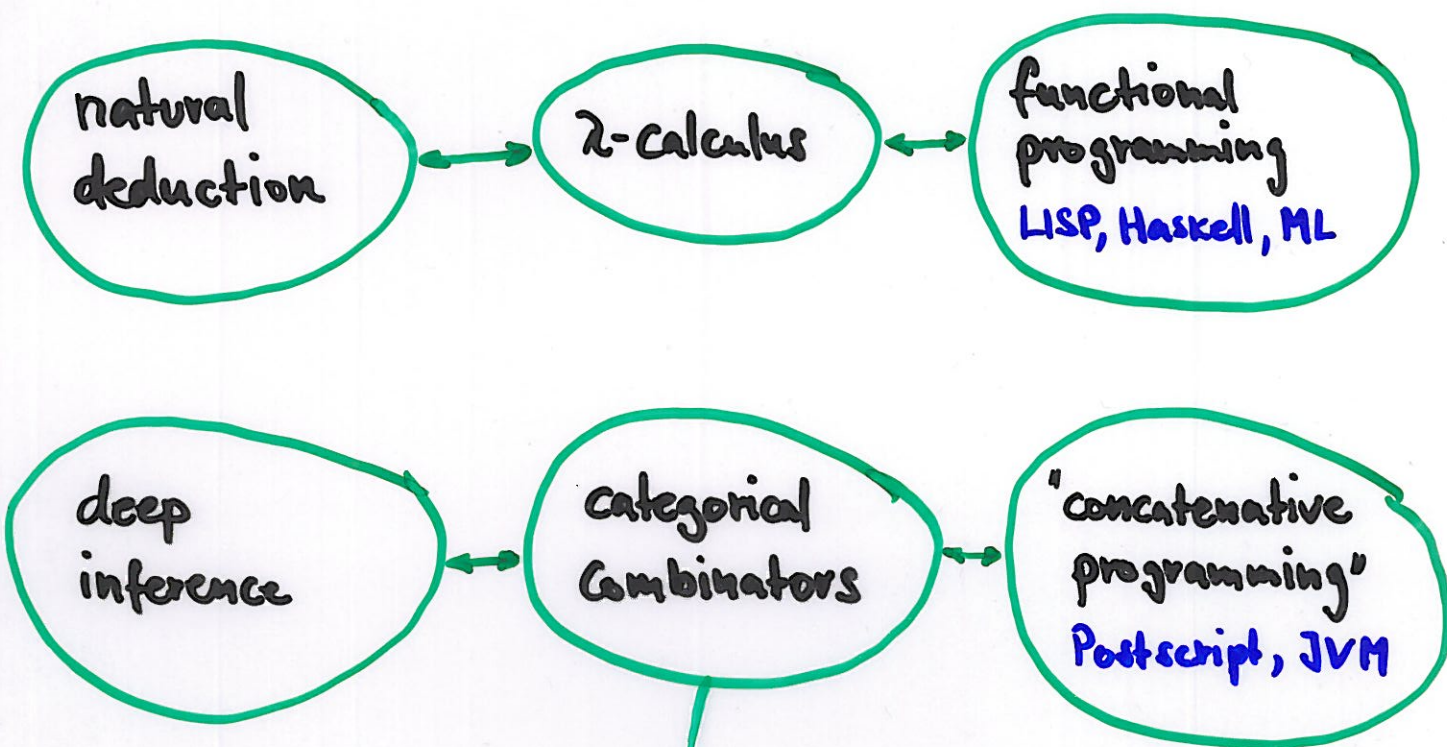


"concatenative programming"
Postscript, JVM

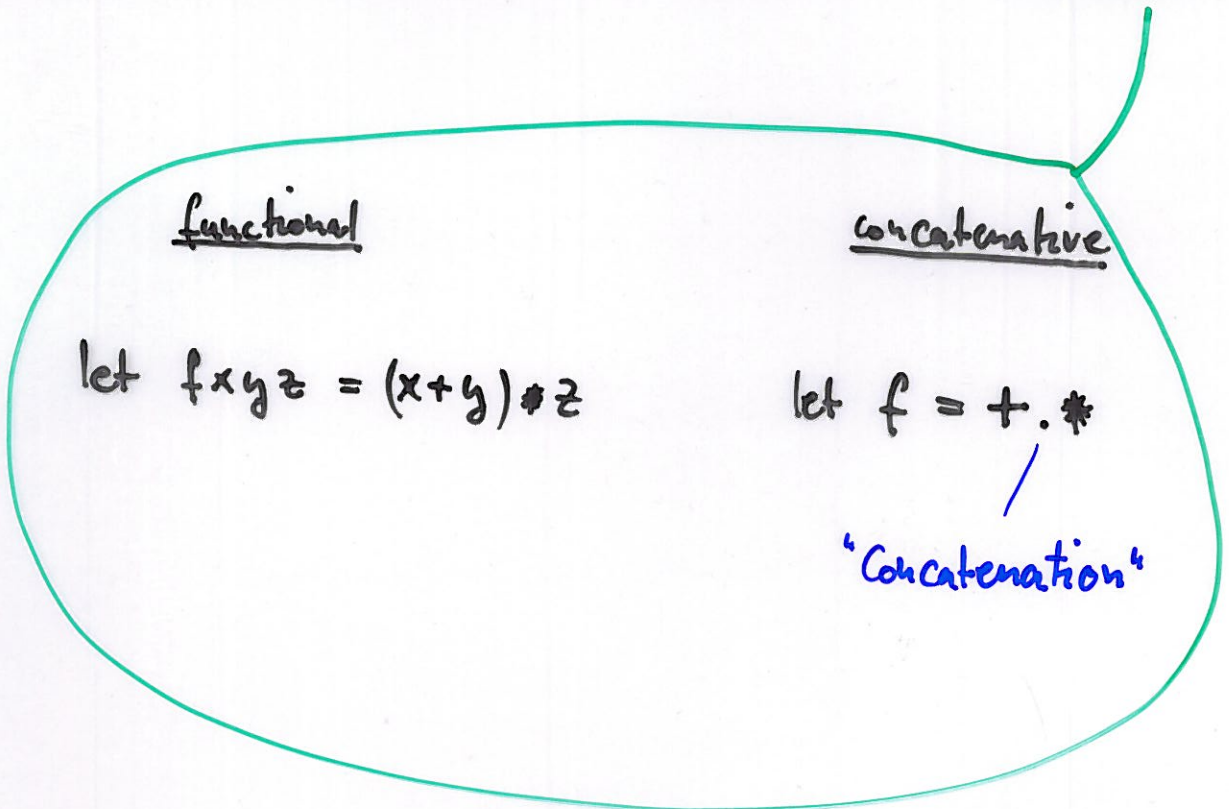
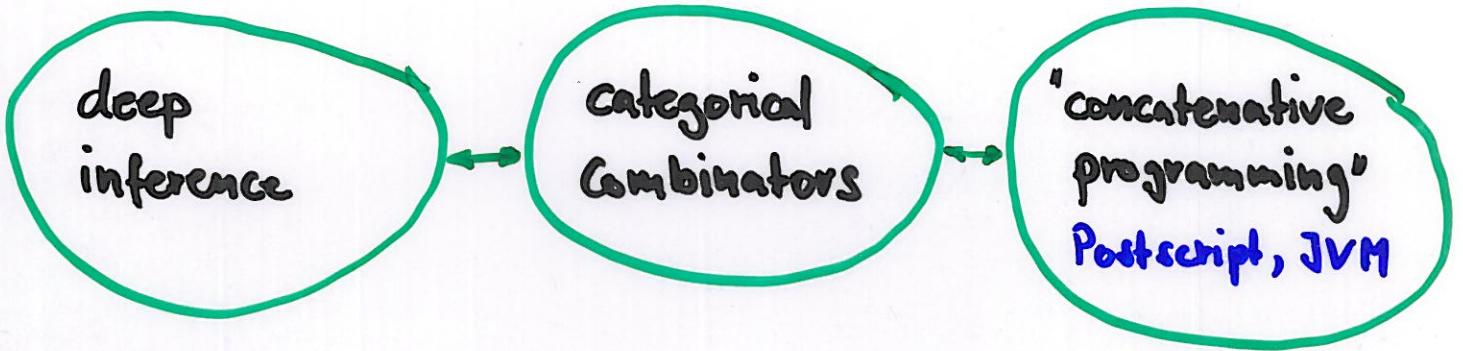
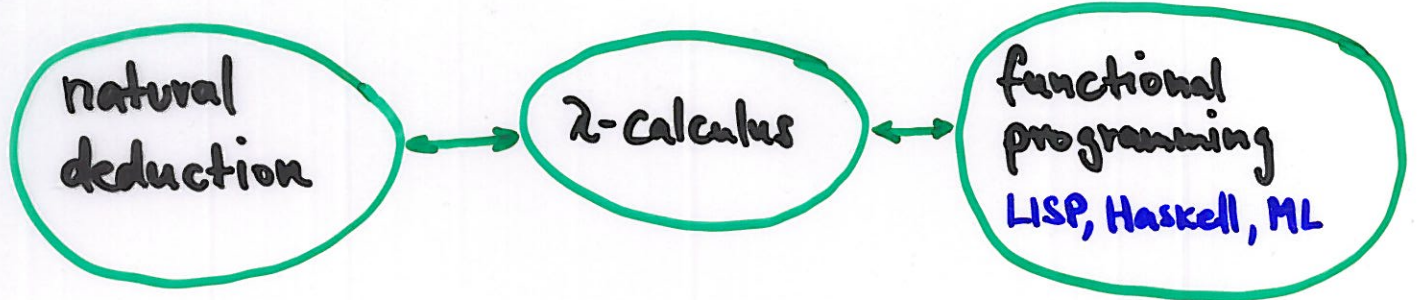
apply $\wedge_E \frac{A \wedge B}{A}$ to $(a \wedge b) \wedge c$

the only possibility in natural deduction: $\wedge_E \frac{(a \wedge b) \wedge c}{a \wedge b}$

also possible in deep inference: $\wedge_E \frac{(a \wedge b) \wedge c}{a \wedge c}$



- developed by Curien in the 80s to compile CAML into
- take the equational theory of the free cartesian closed category and orient them
- basis of the first explicit substitution calculus



A DEEP INFERENCE SYSTEM FOR INTUITIONISTIC LOGIC

$$c \frac{A}{A \wedge A}$$

$$i \frac{B}{A \supset (B \wedge A)}$$

$$w_1 \frac{A \wedge B}{A}$$

$$w_2 \frac{A \wedge B}{B}$$

$$e \frac{(A \supset B) \wedge A}{B}$$

EXAMPLE DERIVATION

$$\begin{array}{l}
 c \frac{A \wedge B}{} \\
 (A \wedge B) \wedge (A \wedge B) \\
 w_2 \frac{}{B \wedge (A \wedge B)} \\
 w_1 \frac{}{B \wedge A}
 \end{array}$$

PROOF TERMS

$$R ::= \text{id} \mid p \in \{c, w_1, w_2, i, e\} \mid R \circ R \mid R \wedge R \mid R \supset R$$

EXAMPLE TERM

$$c \circ (w_2 \wedge \text{id}) \circ (\text{id} \wedge w_1)$$

TYPING RULES

$$\boxed{\begin{array}{c} A \\ \text{id} \\ A \end{array}}$$

$$\boxed{\begin{array}{c} A \\ p \\ B \end{array}}$$

$$\text{if } p \frac{A}{B}$$

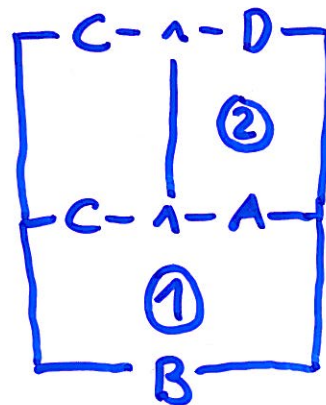
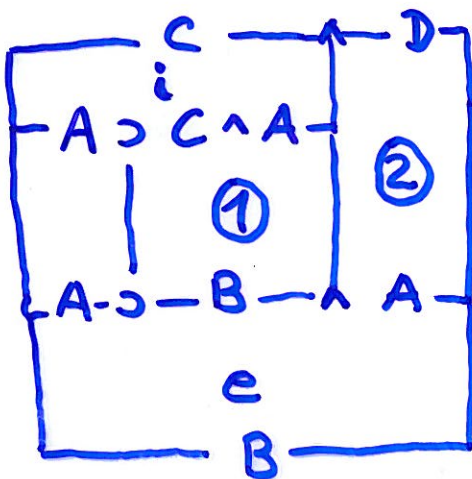
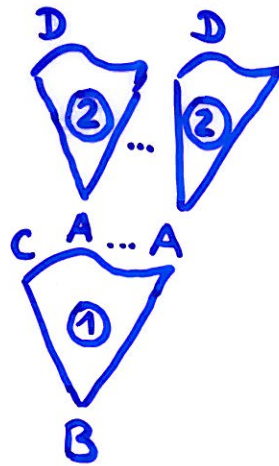
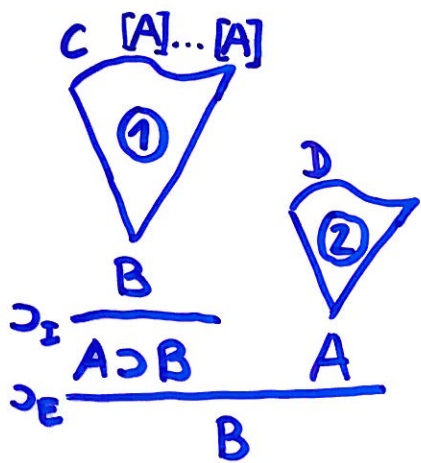
$$\boxed{\begin{array}{c} A \\ R \\ B \\ T \\ C \end{array}}$$

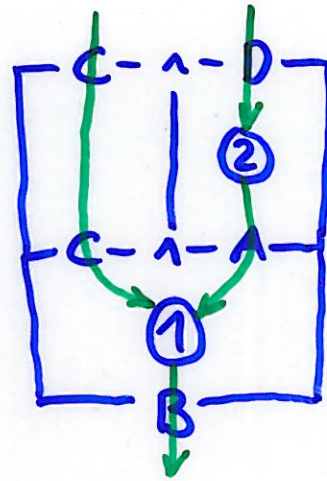
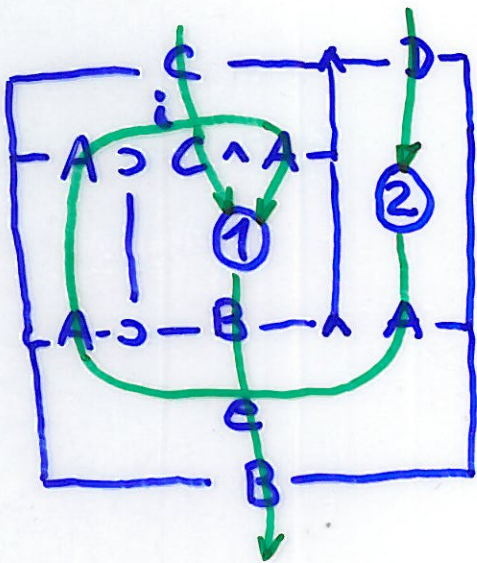
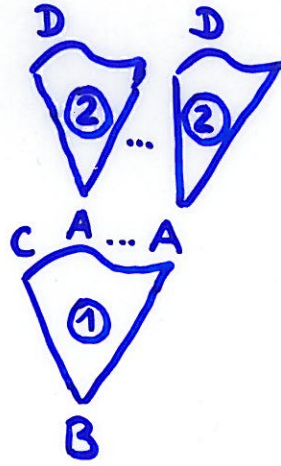
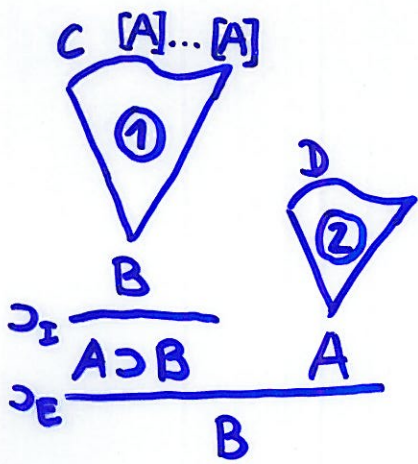
$$\boxed{\begin{array}{c|c} A \wedge C \\ R & T \\ \hline B \wedge D \end{array}}$$

$$\boxed{\begin{array}{c|c} A \supset C \\ \gamma & T \\ \hline B \supset D \end{array}}$$

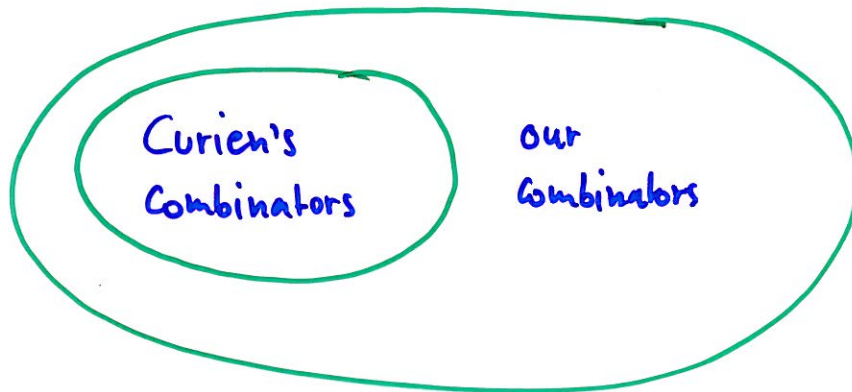
TYPING EXAMPLE

$$\boxed{\begin{array}{c} A \wedge B \\ C \\ \hline A \wedge B \wedge A \wedge B \\ \hline w_2 \quad | \quad \text{id} \\ B \wedge A \wedge B \\ \hline \text{id} \quad | \quad w_1 \\ B \wedge A \end{array}}$$





PROPERTIES OF THE COMBINATORS



	Curien	us
local confluence	NO	YES
Confluence	NO	?
weak normalisation	YES	?
Strong normalisation	NO	?